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Effective energy–momentum tensor of strong-field QED with unstable vacuum

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Abstract
We study the influence of a vacuum instability on the effective energy–momentum tensor (EMT) of QED, in the presence of a quasi-constant external electric field, by means of the relevant Green functions. In the case when the initial vacuum, $|0, \text{in}\rangle$, differs essentially from the final vacuum, $|0, \text{out}\rangle$, we find explicitly and compare both the vacuum average value of EMT, $\langle 0, \text{in}|T_{\mu\nu}|0, \text{in}\rangle$, and the matrix element, $\langle 0, \text{out}|T_{\mu\nu}|0, \text{in}\rangle$. In the course of the calculation, we solve the problem of the special divergences connected with infinite time T of action of the constant electric field. The EMT of a pair created by an electric field from the initial vacuum is presented. The relations of the obtained expressions to Euler–Heisenberg's effective action are established.

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1. Introduction

Currently, an effective action method, originating with Euler and Heisenberg's one-loop effective action, is one of the commonly used approaches of QFT. Nevertheless, we can see that if an external electric field is involved then naive calculations by analogy with a magnetic field case can be erroneous. For example, thermally influenced pair production in a constant electric field has been searched via several attempts at generalization for one-loop effective action at a finite temperature with extremely contrary results. We would like to express that the vacuum instability in an electric background opens additional channels of interaction due to particle creation from the vacuum. That is the reason why the above-mentioned simple analogy does not work, regardless of thermal influence, and we have to refine the set up of the problem. For simplicity of explanation in this talk, we suppose that the temperature is equal to zero[1].

[1] We will present the relevant generalization for one-loop effects at finite temperature anywhere.

The relevant intense field method, applicable to the theory with unstable vacuum in the case of a time-varying external field (called the generalized Furry representation), can be found in [2][2]. Following this method we see that the effective perturbation theory with respect to the radiative interaction for the matrix elements of the scattering processes and another one for the expectation values differ by the type of the one-particle Green function due to the nontrivial difference between a final vacuum, $|0, \text{out}\rangle$, and an initial vacuum, $|0, \text{in}\rangle$, $c_v = \langle 0, \text{out}|0, \text{in}\rangle$, $|c_v|^2 \neq 1$. Feynman diagrams for the matrix elements of the scattering processes have to be calculated by means of the causal propagator

$$S^c(x, x') = c_v^{-1} \mathrm{i}\langle 0, \text{out}|T\psi(x)\bar{\psi}(x')|0, \text{in}\rangle, \tag{1}$$

where $\psi(x)$ is a massive (m) quantum spinor field satisfying the Dirac equation with an external field. In the calculation of the expectation values one has to use the one-particle Green functions

$$\begin{aligned} S^c_{\text{in}}(x, x') &= \mathrm{i}\langle 0, \text{in}|T\psi(x)\bar{\psi}(x')|0, \text{in}\rangle, \\ S^c_{\text{out}}(x, x') &= \mathrm{i}\langle 0, \text{out}|T\psi(x)\bar{\psi}(x')|0, \text{out}\rangle. \end{aligned} \tag{2}$$

Both differ from the causal propagator (1). Additionally, these distinct Green functions are used to represent various matrix elements of operators of the current and energy–momentum tensor (EMT), and effective action beginning with zeroth order with respect to the radiative interaction. Euler and Heisenberg's one-loop effective action $Y_{\text{out-in}}$ is related to the causal propagator, $Y_{\text{out-in}} = \mathrm{i}\,\mathrm{Tr}\ln S^c$. Varying $Y_{\text{out-in}}$, given by the Fock–Schwinger proper time representation [1], one gets the following matrix elements of the operators of a current density, j_μ, and EMT, $T_{\mu\nu}$, in the one-loop approximation:

$$\langle j_\mu\rangle^c = \langle 0, \text{out}|j_\mu|0, \text{in}\rangle c_v^{-1}, \qquad \langle T_{\mu\nu}\rangle^c = \langle 0, \text{out}|T_{\mu\nu}|0, \text{in}\rangle c_v^{-1}, \tag{3}$$

where the operators j_μ and $T_{\mu\nu}$ are in the generalized Furry representation,

$$\begin{aligned} j_\mu &= \frac{q}{2}[\bar{\psi}(x), \gamma_\mu\psi(x)], \qquad T_{\mu\nu} = \frac{1}{2}\left(T^{can}_{\mu\nu} + T^{can}_{\nu\mu}\right), \\ T^{can}_{\mu\nu} &= \frac{1}{4}\{[\bar{\psi}(x), \gamma_\mu P_\nu\psi(x)] + [P^*_\nu\bar{\psi}(x), \gamma_\mu\psi(x)]\}, \\ P_\mu &= \mathrm{i}\partial_\mu - qA_\mu(x), \qquad q = -e. \end{aligned} \tag{4}$$

On the other hand, at a time instant x^0 the average values of j_μ and $T_{\mu\nu}$ operators in the one-loop approximation are the following:

$$\langle j_\mu\rangle^{\text{in}} = \langle 0, \text{in}|j_\mu|0, \text{in}\rangle, \qquad \langle T_{\mu\nu}\rangle^{\text{in}} = \langle 0, \text{in}|T_{\mu\nu}|0, \text{in}\rangle. \tag{5}$$

The equalities $\langle j_\mu\rangle^{\text{in}} = \langle j_\mu\rangle^c$ and $\langle T_{\mu\nu}\rangle^{\text{in}} = \langle T_{\mu\nu}\rangle^c$ hold strictly for theory with stable vacuum. Thus, the well-known explicit expression of $Y_{\text{out-in}}$ [1] is useless for the calculation of any average values and we see it is desirable to find the relevant one-loop effective description for QED with a constant uniform electromagnetic field.

2. Proper time representation

To see the difference between S^c_{in} and S^c, explicitly one can express these functions via the sets of the appropriate solutions of the Dirac equation in an external field (see details in [2]). First, we need two complete and orthonormal sets of the in/out-solutions of the Dirac equation, $\{_\pm\psi_n(x)\}/\{^\pm\psi_n(x)\}$. They describe particles (+) and antiparticles (−) at the initial/final time

[2] The extension of such an approach for finite temperature QED was presented in [3].

instant $x^0_{\rm in}/x^0_{\rm out}$. Second, we find decomposition coefficients $G({}_\zeta|{}^{\zeta'})$ of the out-solutions in the in-solutions[3],

$$^\zeta\psi(x) = {}_+\psi(x)G({}_+|{}^\zeta) + {}_-\psi(x)G({}_-|{}^\zeta). \tag{6}$$

Then from (1), we get the Feynman definition

$$\begin{aligned} S^c(x,x') &= \theta(x_0 - x_0')S^-(x,x') - \theta(x_0' - x_0)S^+(x,x'), \\ S^-(x,x') &= \mathrm{i}\sum_{n,m} {}^+\psi_n(x)G({}_+|{}^+)^{-1}_{nm}\,{}^+\bar\psi_m(x'), \\ S^+(x,x') &= \mathrm{i}\sum_{n,m} {}_-\psi_n(x)[G({}_-|{}^-)^{-1}]^*_{nm}\,{}^-\bar\psi_m(x'), \end{aligned} \tag{7}$$

and for $S^c_{\rm in}$, we have

$$\begin{aligned} S^c_{\rm in}(x,x') &= \theta(x_0 - x_0')S^-_{\rm in}(x,x') - \theta(x_0' - x_0)S^+_{\rm in}(x,x'), \\ S^\mp_{\rm in}(x,x') &= \mathrm{i}\sum_n {}_\pm\psi_n(x)\,{}_\pm\bar\psi_n(x'). \end{aligned} \tag{8}$$

Then one can express the difference as follows:

$$\begin{aligned} S^a(x,x') &= S^c(x,x') - S^c_{\rm in}(x,x'), \\ S^a(x,x') &= -\mathrm{i}\sum_{nm} {}_-\psi_n(x)[G({}_+|{}^-)G({}_-|{}^-)^{-1}]^\dagger_{nm}\,{}_+\bar\psi_m(x'), \end{aligned} \tag{9}$$

and the similar expression can be written for $S^p(x,x') = S^c(x,x') - S^c_{\rm out}(x,x')$. Only if the vacuum is stable then all the coefficients $G({}_+|{}^-)$, and then S^a, S^p are equal to zero.

We consider the general case of a constant uniform electromagnetic field, $F_{\mu\nu}$, with nonzero invariants where an electric field is given by the time-dependent potential. For simplicity, we choose the reference frame in which the electric, **E**, and magnetic, **B**, fields are parallel and directed along the x^3 axis.

All the singular functions in a constant uniform electromagnetic field can be represented [6] as the following Fock–Schwinger proper time integrals:

$$\begin{aligned} &S^{c,a,p}(x,x') = (\gamma P + m)\Delta^{c,a,p}(x,x'), \\ &\Delta^c(x,x') = \int_{\Gamma_c} f(x,x',s)\,\mathrm{d}s = \int_0^\infty f(x,x',s)\,\mathrm{d}s, \\ &\Delta^{a/p}(x,x') = \frac{1}{2}\Delta^{\Gamma_2}(x,x') + \Delta^{\bar a/\bar p}(x,x'), \\ &\Delta^{\Gamma_2}(x,x') = \int_{\Gamma_2} f(x,x',s)\,\mathrm{d}s, \\ &\Delta^{\bar a/\bar p}(x,x') = \left[\Theta(\pm y_3) - \frac{1}{2}\right]\int_{\Gamma_2} f(x,x',s)\,\mathrm{d}s + \int_{\Gamma_a} f(x,x',s)\,\mathrm{d}s \\ &\qquad + \Theta(\pm y_3)\int_{\Gamma_3 - \Gamma_a} f(x,x',s)\,\mathrm{d}s, \qquad y_3 = x_3 - x_3' \end{aligned} \tag{10}$$

where $f(x,x',s)$ is the known Fock–Schwinger proper time kernel [1] and all the contours of the integrals are shown in figure 1. The contours Γ_c and Γ_1 are placed below the singular points on the real axis everywhere outside the origin. Outside the origin the kernel has only one singular point, $s_1 = -\mathrm{i}\pi/eE$, on the complex region between the line of the contours $\Gamma_c - \Gamma_1$ and the line of the contours $\Gamma_a - \Gamma_3$.

[3] We are using a convention of summation/integration over discrete/continuous repeated indices and a compact notation where all summations/integrations are suppressed, for example $\psi_n G_{nm} = (\psi G)_m$. In addition $\hbar = c = 1$ throughout this paper.

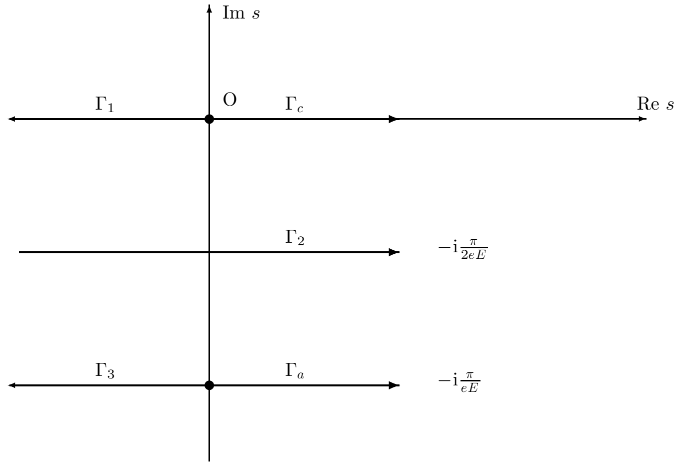

Figure 1. Contours of integration $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_c, \Gamma_a$.

By using these representations, one can uniformly express all the matrix elements of the j_μ and $T_{\mu\nu}$ operators, as follows:

$$\begin{aligned}
&\langle j_\mu\rangle^{\text{in}} = \langle j_\mu\rangle^c - \langle j_\mu\rangle^a, \qquad \langle T_{\mu\nu}\rangle^{\text{in}} = \langle T_{\mu\nu}\rangle^c - \langle T_{\mu\nu}\rangle^a,\\
&\langle j_\mu\rangle^{\text{out}} = \langle j_\mu\rangle^c - \langle j_\mu\rangle^p, \qquad \langle T_{\mu\nu}\rangle^{\text{out}} = \langle T_{\mu\nu}\rangle^c - \langle T_{\mu\nu}\rangle^p,\\
&\langle j_\mu\rangle^{c,a,p} = \mathrm{i}q\, \mathrm{tr}_s\{\gamma_\mu\gamma^\nu P_\nu \Delta^{c,a,p}(x,x')\}|_{x=x'},\\
&\langle T_{\mu\nu}\rangle^{c,a,p} = \mathrm{i}\, \mathrm{tr}_s\{B_{\mu\nu}\Delta^{c,a,p}(x,x')\}|_{x=x'},\\
&B_{\mu\nu} = 1/4\{\gamma_\mu(P_\nu + P'^*_\nu) + \gamma_\nu(P_\mu + P'^*_\mu)\}\gamma^\kappa P_\kappa,\\
&P'^*_\mu = -\mathrm{i}\frac{\partial}{\partial x'^\mu} - qA_\mu(x'),
\end{aligned} \tag{11}$$

where $\mathrm{tr}_s\{\ldots\}$ is the trace of an product of the Dirac gamma matrices.

The expression for the term $\langle j_\mu\rangle^c$ in (11) is finite after the proper time regularization lifting and equal to zero. The components $\langle j_\mu\rangle^{a/p}$ for $\mu \neq 3$ are equal to zero, as well. All the off-diagonal matrix elements of $\langle T_{\mu\nu}\rangle^{c,a,p}$ are equal to zero. It is precisely the term $\langle T_{\mu\nu}\rangle^c$ that can be derived from the Heisenberg–Euler effective Lagrangian, $\mathcal{L}$. Performing the standard renormalizations, leaving $eF_{\mu\nu}$ invariant, one gets the finite expression of $\langle T_{\mu\nu}\rangle^c$ as follows:

$$\begin{aligned}
&\langle T_{00}\rangle^c_{\text{eff}} = -\langle T_{33}\rangle^c_{\text{eff}} = E\frac{\partial\mathcal{L}}{\partial E} - \mathcal{L}, \qquad \langle T_{11}\rangle^c_{\text{eff}} = \langle T_{22}\rangle^c_{\text{eff}} = \mathcal{L},\\
&\mathcal{L} = \int_0^\infty \frac{\mathrm{d}s}{8\pi^2 s}\,\mathrm{e}^{-\mathrm{i}m^2 s}\left[\mathrm{e}^2 EB\coth(eEs)\cot(eBs) - \frac{1}{s^2} - \frac{e^2}{3}(E^2 - B^2)\right].
\end{aligned}$$

Bearing in mind that $\langle T_{\mu\nu}\rangle^{\Gamma_2} = 2\mathrm{i}\,\mathrm{Im}\langle T_{\mu\nu}\rangle^c$, we get for the average values of the operators j_μ and $T_{\mu\nu}$ the following explicitly real expressions:

$$\langle j_\mu\rangle^{\text{in}} = -\langle j_\mu\rangle^{\bar{a}}, \qquad \langle T_{\mu\nu}\rangle^{\text{in}}_{\text{eff}} = \mathrm{Re}\langle T_{\mu\nu}\rangle^c_{\text{eff}} - \langle T_{\mu\nu}\rangle^{\bar{a}}. \tag{12}$$

The terms $\langle j_\mu\rangle^{\bar{a}/p}$ and $\langle T_{\mu\nu}\rangle^{\bar{a}/p}$ are proportional to the factor $\exp\{-\pi m^2/eE\}$. Thus, they are related to global features of the theory and indicate the vacuum instability. These matrix elements are free from the standard ultraviolet divergences. However, with such terms we run into special kind of divergences in the constant electric field due to the contributions from derivatives of $\Theta(\pm y_3)$ functions and singular point s_1. The nature of such special divergences is connected with the infinite time T of action of the constant electric field. They have to be regularized with respect to time T of acting of a constant electric field.

3. Finite work regularization

The state of the quantum system in question is far-from-equilibrium due to the influence of the time-dependent potential of an electric field. Then there exists the problem of time dependence for average values which we discuss here. In a physically correct statement of the problem, we only refer to a quasi-constant electric field which is effectively acting for a finite time T, $E(x^0) = E$ for $t_1 \leqslant x^0 \leqslant t_2$, $t_2 = -t_1 = T/2$, and then does finite work in a finite volume. Out of the time interval T an electric field is absent. Furthermore, we call it T-constant field. In this case the initial vacuum is the vacuum of free particles. General aspects of the special regularization with respect to time T by using the T-constant field was discussed in [4]. Now, we need to apply those results for calculating the leading terms in $\langle j_3\rangle^{a/p}$ and $\langle T_{\mu\nu}\rangle^{a/p}$ at $T \to \infty$.

The mean number of particles created by the external field from the initial vacuum is

$$R_n^{\rm cr} = \langle 0, {\rm in}|a_n^\dagger({\rm out})a_n({\rm out})|0, {\rm in}\rangle = |G({}_-|{}^+)|^2, \tag{13}$$

where the standard volume regularization was used, so that $\delta(\mathbf{p} - \mathbf{p}') \to \delta_{\mathbf{p},\mathbf{p}'}$. If the time T is sufficiently large: $T \gg T_0$, where $T_0 = (1+\lambda)/\sqrt{eE}$ is called the stabilization time, and $eET/2 \gg |p_3|$, then

$$\begin{aligned} &R_n^{\rm cr} = {\rm e}^{-\pi\lambda}\left[1 + O\left(\left[\frac{1+\lambda}{\xi_{\pm 1}}\right]^3\right)\right], \qquad -\sqrt{eE}\frac{T}{2} \leqslant \xi_1 \leqslant -K, \\ &\lambda = \frac{m^2 + \langle P_\perp^2\rangle}{eE}, \qquad P_\perp = (P^1, P^2, 0), \qquad \xi_{\pm 1} = (\mp eET/2 - p_3)/\sqrt{eE}, \end{aligned} \tag{14}$$

where K is a sufficiently large arbitrary constant, $K \gg 1+\lambda$, $\langle P_\perp^2\rangle$ is the conserved average value of $P_\perp^2$, and p_3 is longitudinal momentum. The $R_n^{\rm cr}$ distribution for large longitudinal momenta, $|p_3| \gg eET/2$, decreases, $R_n^{\rm cr} = O\big(\big[\lambda/\xi_1^2\big]^3\big)$. The latter expression allows one to consider the limit $T \to \infty$ at any given quantum number. In this limit, the distribution function takes the simple form $R_n^{\rm cr} = {\rm e}^{-\pi\lambda}$ which coincides with the expressions obtained in the constant electric field [5].

The distribution $R_m^{\rm cr}$ plays the role of the cutoff factor for the integral (9) and similar representation of S^p, then the contributions of $S^{a/p}$ are convergent. If the time interval $x^0 - t_1 = x^0 + T/2$ is sufficiently large, $\sqrt{eE}(x^0 + T/2) \gg 1 + m^2/eE$, we can extract the leading contributions at large T (marked as a subscript 'as') in the representations (11), (12) and then, integrating over quantum numbers and calculating derivatives, obtain that

$$\begin{aligned} &\langle j_\mu\rangle_{as}^{a/p} = -\delta_\mu^3 2e(1/2 \pm x^0/T)n^{\rm cr}, \\ &\langle T_{00}\rangle_{as}^{a/p} = \langle T_{33}\rangle_{as}^{a/p} = -eET(1/2 \pm x^0/T)^2 n^{\rm cr}, \\ &\langle T_{11}\rangle_{as}^{a/p} = \langle T_{22}\rangle_{as}^{a/p} \\ &\qquad = \tilde{n}\begin{cases} \mp \ln[\sqrt{eE}(T/2 \pm x^0)] + O(\ln K) & \text{if} \quad \sqrt{eE}(T/2 \pm x^0) > K \\ O(\ln K) & \text{if} \quad \sqrt{eE}(T/2 \pm x^0) \leqslant K \end{cases} \end{aligned} \tag{15}$$

where K is an arbitrary constant, $K \gg 1 + m^2/eE$,

$$n^{\mathrm{cr}} = \frac{e^2 EBT}{4\pi^2} \coth \frac{\pi B}{E} \left[\exp\left\{ -\pi \frac{m^2}{eE} \right\} + O\left(\frac{K}{\sqrt{eE}T} \right) \right],$$
$$\tilde{n} = \frac{e^2 B^2}{4\pi^2 \sinh^2(\pi B/E)} \exp\left\{ -\pi \frac{m^2}{eE} \right\}. \tag{16}$$

Note that here n^{cr} is a characteristic number density of excitable states in the external field and, as we see subsequently, it is the same as the number density of the created pairs for time T of the duration of the electric field.

The current density and the EMT of the final particles created from vacuum by the T-constant field for the large time interval $x^0 - t_1 = x^0 + T/2 \gg K/\sqrt{eE}$ can be presented as

$$j_\mu^{\mathrm{cr}} = \langle j_\mu \rangle^{\mathrm{in}} - \langle j_\mu \rangle^{\mathrm{out}}, \qquad T_{\mu\nu}^{\mathrm{cr}} = \langle T_{\mu\nu} \rangle^{\mathrm{in}} - \langle T_{\mu\nu} \rangle^{\mathrm{out}}, \tag{17}$$

where the terms $\langle j_\mu \rangle^{\mathrm{out}}$ and $\langle T_{\mu\nu} \rangle^{\mathrm{out}}$ are used to take into account the normal ordering of the current density and the EMT operators with respect to the creation and annihilation operators of the final particles. Then one gets from (11) and (15) that

$$j_\mu^{\mathrm{cr}} = \langle j_\mu \rangle_{as}^p - \langle j_\mu \rangle_{as}^a = \delta_\mu^3 2e(2x^0/T) n^{\mathrm{cr}},$$
$$T_{\mu\nu}^{\mathrm{cr}} = \langle T_{\mu\nu} \rangle_{as}^p - \langle T_{\mu\nu} \rangle_{as}^a, \tag{18}$$

and

$$\langle T_{00} \rangle^{\mathrm{cr}} = \langle T_{33} \rangle^{\mathrm{cr}} = 2eEx^0 n^{\mathrm{cr}},$$
$$\langle T_{11} \rangle^{\mathrm{cr}} = \langle T_{22} \rangle^{\mathrm{cr}}$$
$$= \tilde{n} \begin{cases} \ln[eE((T/2)^2 - (x^0)^2)] + O(\ln K) & \text{if} \quad \sqrt{eE}(T/2 - x^0) \rangle K \\ \ln[\sqrt{eE}(T/2 + x^0)] + O(\ln K) & \text{if} \quad \sqrt{eE}(T/2 - x^0) \leqslant K. \end{cases} \tag{19}$$

At $x_0 = t_2 = T/2$ one gets from (18), (19) the expressions for the total current densities and the EMT of the particles created.

4. Conclusion

We finally obtain the average values of the current density and the EMT as follows:

$$\langle j_\mu \rangle^{\mathrm{in}} = -\langle j_\mu \rangle_{as}^a, \qquad \langle T_{\mu\nu} \rangle_{\mathrm{eff}}^{\mathrm{in}} = \mathrm{Re} \langle T_{\mu\nu} \rangle_{\mathrm{eff}}^c - \langle T_{\mu\nu} \rangle_{as}^a. \tag{20}$$

As we have seen, the T-dependent contributions to $\langle j_\mu \rangle^{\bar{a}}$ and $\langle T_{\mu\nu} \rangle^{\bar{a}}$ appear due to the vacuum instability and then come with the factor $\exp\{-\pi m^2/eE\}$. This factor is exponentially small for a weak electric field, $m^2/eE \gg 1$, and the effect can actually be observed as soon as the external field strength approaches the characteristic value $E_c = m^2/e$. On the other hand, the term $\mathrm{Re}\langle T_{\mu\nu} \rangle_{\mathrm{eff}}^c$ is T-independent and its contribution is not small whether the electric field is weak or strong. When the T-constant electric field is switched off at $x^0 > T/2$, the local vacuum contribution of E in $\mathrm{Re}\langle T_{\mu\nu} \rangle_{\mathrm{eff}}^c$ is absent but the global contribution given by $\langle T_{\mu\nu} \rangle_{as}^a \big|_{x^0 = T/2}$ is present. Thus, in the general case both kinds of contributions are important.

Using expression (20), we find a condition for validity of a strong constant electric field concept. With a very strong E field, $m^2/eE \ll 1$ $(B = 0)$, and large T one gets the well-known asymptotic expression of the $\mathrm{Re}\langle T_{00} \rangle_{\mathrm{eff}}^c$ vacuum energy density

$$\mathrm{Re}\langle T_{00} \rangle_{\mathrm{eff}}^c = -\frac{e^2}{24\pi^2} E^2 \ln \frac{eE}{m^2}.$$

It is T-independent contribution. The energy density of a classic electric field is $E^2/8\pi$. Then it seems that an electric field concept is physically meaningful when $\frac{e^2}{3\pi} \ln \frac{eE}{m^2} \ll 1$. But when

T is large one has to give attention to the T-dependent term $\langle T_{\mu\nu}\rangle^a_{as}$. At $x^0 = T/2$, we have

$$-\langle T_{\mu\nu}\rangle^a_{as} = \frac{e^2E^2}{4\pi^3}eET^2.$$

Of course, one can neglect a back-reaction on an electric field only if the last term is far less than $E^2/8\pi$. Thus, the true condition for validity of a strong constant electric field concept is the following:

$$1 \ll eET^2 \ll \frac{\pi^2}{2e^2}.$$

All the results for pair creation are valid within the accuracy of the analysis at low density and temperature, $\Theta \ll eET$.

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